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Features of Parameters Identification of Algebraic Mathematical Models

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Abstract* — *The conditions related to algorithms of identification of parameters of mathematical models of physical processes in algebraic form is discussed in the paper. The features of these algorithms were shown. One possible approach for solutions this problem was suggested in determinate statement. On real observations were obtained calculations economical characteristics of Ukraine as example.

***Index Terms*— algebraic mathematical models, the features, identification of parameters, regularization.**

I. INTRODUCTION

Mathematical modeling of physical processes is an essential tool for environmental studies [1],[2].

The successful mathematical modeling needs to have a mathematical model of the physical process (MM) and a model of external influence on the process. In turn, the mathematical model of the physical process can be in the form of ordinary differential equations or systems of equations partial derivatives of algebraic relations, integral equations, and others. For example, the simplest mathematical model of reality is a number.

In many cases, a mathematical model is constructed as an algebraic relation between the characteristics of the physical process. This type of models are widespread in electromagnetic, econometrics, eg, [1], [2]. For a reasonable application of mathematical models of the specified type should be taken into account that they are receiving under certain conditions. Let us examine these conditions in more detail.

II. PROPERTIES OF PHYSICAL PROCESSES

Let the physical processes at the initial time $t = t_0$ is characterized, in general, infinite number of variables $q_1, q_2, ..., q_n, ...$ (characteristics). Then, these variables are transformed during time according to some algorithm and we get some indicators of physical processes $u_1, u_2, ..., u_k$ at time $t = T$. Performance objectives $u_1, u_2, ..., u_k$ are defined physical process studies. If the algorithm of conversion time does not change, then it is possible to obtain the relationship between the variables $q_1, q_2, ..., q_n, ...$ and parameters $u_1, u_2, ..., u_k$. In general, this relationship is a complex nonlinear function of the infinite set of variables:

$$\bar{\varphi}(q_1, q_2, ..., q_n, ...) = (\varphi_1(q_1, q_2, ..., q_n, ...), \ldots, \varphi_n(q_1, q_2, ..., q_n, ...))^T =$$
$$= \bar{u} = (u_1, \ldots, u_k)^T, \qquad (1)$$

where $(.)^T$ is the transpose operation.

If only some of the characteristics $q_1, q_2, ..., q_n$ of the process of change over time, and the latter almost no change or a major impact on the performance characteristics $u_1, u_2, ..., u_k$ render only $q_1, q_2, ..., q_n$ then the relationship (1) is converted to the following approximate

$$\tilde{\bar{\varphi}}(q_1, .., q_n) = (\tilde{\varphi}_1(q_1, .., q_n), \ldots, \tilde{\varphi}_n(q_1, .., q_n))^T =$$
$$= \tilde{\bar{u}} = (\tilde{u}_1, \ldots, \tilde{u}_k)^T . \qquad (2)$$

If we limit ourselves to only small changes in the characteristics $q_1, q_2, ..., q_n$ of the area D around the point $(q_1^0, q_2^0, ..., q_m^0)$, dear then function $\tilde{\varphi}_k(q_1, q_2, ..., q_n)$ in equation (2) may change approximately linear relationship:

$$A\bar{q} = \tilde{\bar{u}}, \ \bar{q} = (q_1, q_2, \ldots, q_n)^T, \qquad (3)$$

where A is the matrix of linear mathematical models of communication vector $\bar{q}$ with the vector $\tilde{\bar{u}}$ size $k \times n$.

Let us consider one term in (3):

$$a_{k,1} q_1 + a_{k,2} q_2 + ... + a_{k,n} q_n = \tilde{u}_k \quad , \qquad (4)$$

where $a_{k,1}, a_{k,2}, \ldots, a_{k,n}$ the parameters of the approximate mathematical model of physical communication process $q_1, q_2, ..., q_n$ with the index $\tilde{u}_k$.

Since the physical process is the same time, the parameters of the mathematical model of the physical process will be constant [3,4]. We will further linear mathematical model (3) is called a local mathematical model in the neighborhood of the point $(q_1^0, q_2^0, ..., q_n^0)$. In addition, the process of constructing a mathematical model (3) may conclude that there is, in principle, a mathematical model of the type (3), which would accurately describe the connection parameters of the real physical process. This physical process may be called stable process in view of the constancy of algorithm

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converting of variables. Suppose there are experimental measurements $\tilde{\tilde{q}}=(\tilde{q}_1,\tilde{q}_2,...,\tilde{q}_n)^T$ with maximum accuracy δ_0. If the substitution of these measurements $\tilde{q}_1,\tilde{q}_2,...,\tilde{q}_n$ in (3) we get $A\tilde{\tilde{q}}$ that rate different from the measurement $\tilde{\tilde{u}}$ at the lowest value δ that does not exceed the error δ_0 of measurement $\tilde{\tilde{u}}$, while the mathematical model (3) will be called *adequate mathematical model* (or *adequate mathematical description*) of physical processes. If the inequality $\delta_0>\delta$ for parameters $\tilde{u}_k$, then a mathematical model (4) will be called conditionally with adequate accuracy δ by index $\tilde{u}_k$.

Based on the conditions of constructing approximate mathematical models of physical processes in algebraic form may formulate the following constraints on physical processes:

1. Algorithm for converting variables $\bar{q}=(q_1,q_2,...,q_n)^T$ to a final value $\tilde{\tilde{u}}$ does not change over time (algorithm is stable);

2. Variables $\bar{q}=(q_1,q_2,...,q_n)^T$ are in some small bounded are $D\subset R^n$;

3. The impact $q_{n+1},q_{n+2},...$ on performance indicator $\tilde{\tilde{u}}$ is insignificant or these characteristics do not change during the study of the physical process.

Note some properties of the local linear approximate mathematical models of physical processes in algebraic form:

1). If measurements $\tilde{q}_1,\tilde{q}_2,...,\tilde{q}_n$ substitution in (3) we obtain index $A\tilde{\tilde{q}}$ is different from the measurement $\tilde{\tilde{u}}$ of the smallest value δ that does not exceed δ_0 the error of measurement $\tilde{\tilde{u}}$, while the mathematical model (3) will give *adequate mathematical models of physical processes*;

2). Given the uncertainty of measurement for all the characteristics $\bar{q}=(q_1,q_2,...,q_n)^T$ and indicators $\tilde{\tilde{u}}$ may be argued that there is an infinite number of mathematical models of type (3), which satisfies the condition of adequacy.

3). Results of mathematical models (parameters) to be robust to small changes in initial data.

4). Mathematical models of type (3) for any choice of parameters $a_{k,1},a_{k,2},...,a_{k,n}$ are approximate only.

5). Number of initial data required to be a minimum.

III. FEATURES OF PARAMETERS IDENTIFICATION

Based on the properties of the physical processes that can be represented in algebraic form, you can make some restrictions on the parameters of these algorithms for identification of mathematical models.

First, it is difficult to interpret the results as properties of real physical processes, mathematical models of type (3) are approximate only approximations of reality [2]–[4].

Secondly, it is not meaningful to assess the error of the approximate solution of the problem of identification. This error may be any value. However, this drawback does not prevent the use of the results of parameters identification (mathematical models in algebraic form) to predict the behavior of a physical process, if the initial parameters are changed in some small neighborhood $(q_1^0,q_2^0,...,q_m^0)$. Given the above we can assume that the results of identification of the parameters in the use of data with random errors into measurements cannot produce results which are correspond to adequate mathematical models.

Moreover, the hypothetical exact solution of the problem of identification will be given worse results in predicting force mistakes mathematical description of the physical process [5],[6].

Note that obtaining the mathematical model of the type (5) can be used for the synthesis processes with predetermined characteristics $u_1,u_2,...,u_k$.

IV. POSSIBLE ALGORITHM OF PARAMETERS IDENTIFICATION

We shall present the problem of synthesis of linear mathematical model with n variables relatively q_1 for number of measurements $m=n$, as a problem of the solution of system [7]:

$$A_p(q_2,q_3,...,q_n)z=q_1, \qquad (5)$$

where the operator $A_p(q_2,q_3,...,q_n)z$ is determined as follows

$$A_p(q_2,q_3,...,q_n)z=z_1q_2+z_2q_3+\dots+z_{n-1}q_n+z_ne,$$

e is the unit vector of dimension n.

As the measurements of variables are received experimentally it is assumed that each measurement q_{ij}, $1\le i,j\le n$ has some error the maximal size of which is known:

$$\left|q_{ij}-q_{ij}^{ex}\right|\le\delta_i, 1\le j\le n, 1=1,2,...n. \qquad (6)$$

where q_{ij}^{ex} is exact measurement of variable q_i. The similar information of measurement errors, as a rule, is known a priori. As a rule the maximal error of variable is defined in some percents. The statistical characteristics of errors of measurements are unknown.

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Let us suppose that $z \in R^n; q_i \in R^n; 1 \le i \le n; A_p : R^n \to R^n$, where R^n is Euclidean vector space with standard norm. It is easy to show that the operator A_p is linear. Each vector q_i can accept meanings in some closed area $D_i \subset R^n$ by virtue of inequalities (6).

Let us denoted vector p as vector from space $R^n \otimes R^n \otimes R^n \otimes \ldots R^n = R^{n(n-1)}$:

$$p^* = (q_{21}, q_{22}, \ldots, q_{2n}, q_{31}, \ldots, q_{3n}, \ldots, q_{n1}, \ldots, q_{nn}) .$$

Vectors p can accept meanings in some closed area $D = D_2 \otimes D_3 \otimes D_4 \otimes \ldots \otimes D_n \subset R^{n(n-1)}$. The certain operator A_p associates with each vector p from area D. The class of operators $\{A_p\} = K_A$ will correspond to the set $D \subset R^{n(n-1)}$.

Shall we rewrite (5) as

$$A_p z = u_{\delta_1}, \qquad (7)$$

where $q_1 = u_{\delta_1} \in U = R^n$.

Therefore right part $q_1 = u_{\delta_1}$ and operator $A_p(q_2, q_3, \ldots, q_n)$ in equation (5) are given with an error

$$\left\| u_{\delta_1} - u_1^{ex} \right\| \le \delta_1, \quad \sup_{p_\alpha, p_\beta \in D} \left\| A_{p_\alpha} - A_{p_\beta} \right\|_{Z \to U} \le h_1 ,$$

where u_1^{ex} is exact right part of the equation (5), $\| . \|$ is the norm of a vector in Euclidean space R^n.

Let us consider now the set of the solutions of the equation (5) with the fixed operator $A_p \in K_A$:

$$Q_{\delta_1, p} = \{ z : \left\| A_p z - u_{\delta_1} \right\| \le \delta_1 \}.$$

The set $Q_{\delta_1, p}$ is limited if $\Delta = \det A_p \ne 0$ and unlimited if $\Delta = \det A_p = 0$. Among the operators $A_p \in K_A$ there is at least one operator A_{p_1} with $\Delta = \det A_{p_1} = 0$ with guarantee. However, during synthesis of mathematical model of the physical process in algebraic form is necessary to consider all possible cases of operators in equation (5).

Any vector z from set $Q_{\delta_1, p}$ is the adequate mathematical model of process so this vector after action of the operator A_p coincides with the given vector q_1 with accuracy of measurement δ_1. For choice of particular model from set $Q_{\delta_1, p}$ it is necessary to use additional conditions. If such conditions are absent then it is possible to accept as the solution (5) the element $z \in Q_{\delta_1, p}$ for which the equality is carried out [9],[10]:

$$\left\| z_p \right\|^2 = \inf_{z \in Q_{\delta_1, p}} \| z \|^2 . \qquad (8)$$

The vector $z \in Q_{\delta_1, p}$ is possible to interpret as a maximum steady element to the change of the factors which not taken into account (most stable part), as the influence of these factors will increase norm of a vector z_p [8]. Such a property of the solution z_p is especially important if one takes into account that the vector z_p further will be used for forecasting real processes (parameter q_1).

Consider now the set $Q^* = \bigcup_{p \in D} Q_{\delta_1, p}$ [9].

Let us consider an extreme problem

$$\left\| z^* \right\|^2 = \inf_{p \in D} \inf_{z \in Q_{\delta_1, p}} \| z \|^2 . \qquad (9)$$

The vector $z^* \in Q^*$ is estimation from below of possible solutions of the equation (5). The similar problem in classical identification is not examined.

The statement of the following extreme problem is possible also:

$$\left\| z^*_{\sup} \right\|^2 = \sup_{p \in D} \inf_{z \in Q_{\delta_1, p}} \| z \|^2 . \qquad (10)$$

The vector $z^*_{\sup} \in Q^*$ has the smallest norm among the solutions of a problem of synthesis on sets $Q_{\delta_1, p}$. The vector $z \in Q_{\delta_1, p}$ is possible to interpret as a maximum steady element to the change of functions $q_1 = u_{\delta_1}$ and operators $A_p \in K_A$ (most stable part), as the influence of these factors will increase norm of a vector z_p [8]. The similar problem in the literature is not considered earlier.

Models z^*, $z^*_{\sup}$ can be used for short-term forecasting of change of variable q_1 as on the one hand models z^* and $z^*_{\sup}$ are received by an rapid way and on the other hand these models are steadiest to the change of the factors not taken into account.

Except (9),(10) it is possible to examine the following statements of problems:

$$\left\| z_{0,0,\ldots,1} \right\|^2 = \inf_{q_2 \in D_2} \inf_{q_3 \in D_3} \ldots \inf_{q_{n-1} \in D_{n-1}} \sup_{q_n \in D_n} \inf_{z \in Q_{\delta_1, p}} \| z \|^2 , \qquad (11)$$

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$$\left\| z_{0,0,\ldots,1,1} \right\|^2 = \inf_{q_2 \in D_2} \inf_{q_3 \in D_3} \ldots \sup_{q_{n-1} \in D_{n-1}} \sup_{q_n \in D_n} \inf_{z \in Q_{\delta_1,p}} \| z \|^2, \quad (12)$$

. . . .

$$\left\| z_{0,1,\ldots,1,1} \right\|^2 = \inf_{q_2 \in D_2} \sup_{q_3 \in D_3} \ldots \sup_{q_{n-1} \in D_{n-1}} \sup_{q_n \in D_n} \inf_{z \in Q_{\delta_1,p}} \| z \|^2. \quad (13)$$

In some cases it is expedient to consider the following problems of identification of parameters:

$$\left\| z^{0,0,\ldots,0} \right\|^2 = \inf_{z \in Q_{\delta_1, p^{0,0,\ldots,0}}} \| z \|^2, \quad (14)$$

$$\left\| z^{0,0,\ldots,1} \right\|^2 = \inf_{z \in Q_{\delta_1, p^{0,0,\ldots,1}}} \| z \|^2, \quad (15)$$

. . . .

$$\left\| z^{1,1,\ldots,1} \right\|^2 = \inf_{z \in Q_{\delta_1, p^{1,1,\ldots,1}}} \| z \|^2, \quad (16)$$

where vector $p^{0,0,\ldots,0}$ has the minimal possible size of all components of vector p, $p^{0,0,\ldots,1}$ has the minimal possible size of components $q_2, q_3, \ldots, q_{n-1}$ and has the maximal size of q_n; . . . ; vector $p^{1,1,\ldots,1}$ has the maximal possible size of all components of vector p.

It is possible to consider the following extreme problem

$$\left\| A_{p^{opt}} z_{\delta_1}^{pl} - u_{\delta_1} \right\|^2 = \inf_{z_a \in Q^*} \sup_{A_p \in K_A} \left\| A_p z_a - u_{\delta_1} \right\|^2, \quad (17)$$

where z_α is the solution of extreme problem

$$\| z_a \|^2 = \inf_{z \in Q_{\delta_1,a}} \| z \|^2. \quad (18)$$

Let's called solution $z_{\delta_1}^{pl}$ as ***more plausible mathematical model***.

Use of such model with the purpose of the forecast allows to receive the characteristic q_1 with the least maximal deviation from experiment with possible variations of variables $q_2, q_3, \ldots, q_n$ within the given errors.

Theorem. Solution $z_{\delta_1}^{pl}$ of extreme problem (17) exists, uniquely and is steady to small changes of the initial data if the vector $z_{\delta_1}^{pl}$ is defined uniquely from condition (18) [7].

One of possible ways of the solution of extreme problems (9) - (13),(17) is use in accounts of special mathematical models of researched processes [9],[10]. If the special mathematical models exist then the solution of extreme problems (9) - (13), (17) can are replaced with more simple extreme problems with the precisely given operator such as problems (14) - (16). Further solution is carried out by regularization method of Tikhonov with a choice of regularization parameter by discrepancy method [11].

For approximate solution of extreme problem (17) the interval of change of each components of vector p is divided by a uniform grid p_m. The number of the operators in set will be final $K_A = \{A_1, A_2, .., A_N\} = \{A_i\}$. For each operator A_i the solution z_i is defined. Further the approached solution of an extreme problem (17) is being defined by simple sorting.

$$\left\| A_{p^{opt}} z_{\delta_1}^{pl} - u_{\delta_1} \right\|^2 = \inf_{z_a \in Q^*} \sup_{A_p \in K_A} \left\| A_p z_a - u_{\delta_1} \right\|^2 =$$

$$= \min_j \max_i \{ \left\| A_i z_j - u_{\delta_1} \right\|^2 \}, \ 1 \le i \le m, 1 \le j \le m \ .$$

where z_i z_j satisfies the condition

$$\left\| A_j z_j - u_{\delta_1} \right\|^2 = \delta^2 .$$

V. TEST CALCULATION

The problem of econometric model construction was chosen for test calculation with the use of data which are given on sait of National Statistic administration of Ukraine (Table .I).

At the beginning the econometric model was constructed with application of the least squares method for the first eight years in the assumption that the errors of measurements satisfies to the normal law of distribution [12].

The minimization was carried out with respect to first variable q_8 (on a vertical).

With the using of assumption that the errors of measurements do not surpass 5 %, the mathematical model z_p as result of the solution of the following extreme problem (17) was constructed:

$$q_1 = 0.08 q_2 + 0.41 q_3 + 0.32 q_4 - 0.4 q_5 + \\ +0.5 q_6 - 0.19 q_7 - 0.71 q_8 - 0.53 . \quad (19)$$

For comparison of mathematical models which are received the method of least squares method and by regularization method, the calculations of the characteristic q_1 on the following two years according to these models and comparison of these data with real measurements according to the Table I were made. The results of comparison are presented in the Table II.

The result of real measurements estimate by regularization method [11] shows that mathematical model z_p describes the real situation more exactly than classical mathematical model.

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TABLE I. DATA FOR CALCULATIONS

N/N	q_{1i}	q_{2i2t}	q_{3i3it}	q_{4i4t}	q_{5i}	q_{6i}	q_{7i}	q_{8i}
1	406	130	22.8	11.1	49.1	41.6	34.9	79
2	430	170	28.3	11.5	48.7	56.8	62.4	77
3	469	204	11.9	10.8	48.2	68.4	74.6	74
4	494	226	0.74	9.63	47.8	81.3	85	76
5	541	267	5.21	9.06	47.4	101.5	103.9	79
6	606	345	9.04	8.6	47.1	128	143	85
7	622	441	13.6	7.19	46.8	185	195	78
8	668	544	9.06	6.8	46.5	235	243	81
9	721	721	12.8	6.35	46.2	302	316	89
10	736	948	25.2	6.4	45.9	420	450	189

Data in lines correspond to years:1 – 1999, 2 – 2000, 3 – 2001, 4 – 2002, 5 – 2003, 6 – 2004, 7 – 2005, 8 – 2006, 9 – 2007, 10 – 2008; q_{1i} – gross output of Ukraine in constant price (in billion uaн), q_{2i} – gross output of Ukraine in current price (billion uaн), q_{3i} – inflation rate in per cent, q_{4i} – level of unemployment in per cent, q_{5i} – population (in millions persons), q_{6i} – the income of public administration of Ukraine (in billion uaн), q_{7i} – the expenses of public administration (in billion uaí), q_{8i} – the debt of public administration (in billion uaн).

TABLE II.COMPARISON OF ECONOMIC PARAMETERS`

Years	Tab.I	model obtained by least squares method	model (17)
2007	89.	84.68	90.05
2008	189	95.79	153.12

It is necessary to note that the quality of the mathematical model for the long period is worse, than for the short period. This effect was expected as the offered algorithms were designed for local domain of characteristics change.

Choice in practical problems the certain mathematical model is being determined of the specificity of a concrete problem and final goal of use of mathematical model. However the best model can not be determined a priori.

VI. CONCLUSION

The features of mathematical models of physical processes in algebraic form are considered. It is shown that these features imposed certain restrictions on the possible algorithms of parameters identification of these mathematical models. The possible algorithm of parameter identification with account of these features is suggested. The offered approach to a problem of identification of parameters of mathematical model in algebraic form allows expand a class of the possible solutions (mathematical models) up to maximal possible. Different variants of statement of such problems of parameters identification are suggested.

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